sistently omitting first derivatives of *all* energy con-
tions. It is commonly presumed[25] that the second
ivative E_0'' is small because the actual equilibrium
larger than the value of r for the minimum of E_0.
ilibrium r then occurs in the neighborhood of the
ction point of E_0, as is shown by the available
ulations for copper[26] and silver[27] and by the analyti-
approximation given above. The contribution of
to the bulk modulus and its hydrostatic strain
ivative is probably small therefore. The third
ivative E_0''' contributes to the hydrostatic strain
ivative only; it is felt that it is also likely to be
all in view of the fact that E_0''' is zero at a value of
st beyond the inflection point of E_0, according to the
lytical approximation, and hence also near the
ilibrium value of r. Quantitative estimates of the
sible values of $\Omega B_0=r^2E_0''/9$ and $\Omega dB_0/d\ln r$
$r^2E_0'''-3r^2E_0'')/9$ can be made by using the analyti-
approximation, equating br^{-1} to the Coulomb energy
the structure[22] and invoking the physical condition
$r(\text{equilibrium})>r(E_0'=0)$; these support the state-
nts that have been made, the possible fractional
or in the hydrostatic strain derivative being negli-
ble while those in the bulk modulus may be significant
t are not serious to the conclusions of this paper.

The long-range bulk modulus which has been used
e is then the Fermi term only, and furthermore for
s term we have used an effective mass, m^*/m, of
ty for all three metals. This value of the effective
ss agrees with the theoretical values of Kambe[28]
ich characterize the electrons at the bottom of the
lence band for copper, silver, and gold. It also
rees with electronic specific heat effective masses[29] for
ver and gold, but not for copper in which this m^*/m
1.47. We feel, however, that a "bulk modulus effective
ss," which characterizes the change with volume of
average Fermi energy, is more likely to be equal to
theoretical value than to an effective mass de-
bing the density of states at the Fermi level only.[29]
e have therefore used unity for copper also.

As mentioned above, the long-range contributions to
shear stiffnesses which have been used are the
ulomb stiffnesses of Fuchs, and these have been
en at their full value. Since these terms have been
en at reduced values in other papers[2,3] in which
tic constants have been decomposed into contribu-
ns, we state our reasons. In the first place, the
chs values have long been known to account for the
shear stiffnesses of bcc Na and K,[21] and recently this has been found[30] to be true in Li also. In the alkali metals the long-range term is the major if not the only one and the agreement argues for the validity of the Fuchs calculation. There is no direct evidence for such a long-range stiffness in copper, silver, and gold but extensive studies of the elastic constants of copper[31] and silver alloys[17] in our laboratories provide good indirect evidence. The alloy results require that sizable long-range and short-range terms must both be present, and that C/C' (long range) must be about the Fuchs ratio. These two reasons lead us to regard the Fuchs values as very reasonable estimates of the long-range shear stiffness.

In some previous decompositions of elastic stiffnesses into contributions a van der Waals term has been introduced explicitly.[26] We have omitted such a term as we feel it adds nothing to the analysis which has been carried through and is a numerically uncertain contribution at best. If a contribution to the total energy of the physical nature of the van der Waals interaction is present, it is absorbed, in our treatment, in the short-range repulsive interaction $W=A\exp(-pr/r_0)$ which we have deduced empirically. Formally the van der Waals interaction is radial and of short range and cannot be separated empirically from the repulsive term.

The uncertainties in the analysis presented in the previous section thus reside almost entirely in the theoretical long-range terms. We emphasize again that these terms are small and even large individual errors would leave the conclusions unchanged. The cumulative effect of these uncertainties added to the experimental error, particularly in B and dB/dP, could be considerable, however, so that the individual numerical values of the closure failures which have been quoted and assigned to noncentral short-range interaction should be treated with caution. Nevertheless the relative values of the closure failures appear to be reasonable for the two shear constants and for the three metals.

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